Proposal of a Dynamic Systematic Portfolio Optimization: a development from the Behavioral Finance.

Valéria Saturnino[1], Pierre Lucena[2], Josete Santos[3], Odilon Saturnino[4]

ABSTRACT

The general objective of this research is to model a systematic optimization of portfolios that accompanies the dynamism of the herd effect and the overconfidence, if these are present in the Brazilian capital market.

To this objective, an empirical application was made, and for this the universe considered is the database of stocks traded on the BM&FBOVESPA (Brazilian Stock Exchange), from 1995 to 2015, accessed through the system Economática. Winning and losing portfolios were created in the Brazilian capital market - composed of the 15 stocks with the highest returns in the year of formation, and the 15 stocks with the lowest returns in the year of formation, respectively - and analyzed their returns over the next five years. Moreover, was created a recommended portfolio, based on fundamentalist indexes, to be the basis for the application of the systematic proposal.

The proposal of the monitoring system consists in the definition of a stock scoring criterion to select the 15 stocks that will be part of the recommended portfolio to be monitored and then optimize it. This criterion of scoring is divided into two parts, the first of which refers to the creation of a measure that indicates a fundamentalist advantage of investing in that stock, while the second measure is to evaluate the whole context together, also considering the behavioral variables, according to scenarios described later. In relation to the score that will indicate a fundamentalist advantage of investing in that stock, the four indicators considered were those present in the Five-Factor Model of Fama and French (2015), according to the expectations predicted by the authors.

To identify the herd effect, it has previously been modeled by Chang, Cheng, and Khorana (2000), being the effect measured from the standard deviation of returns in relation to the market average in a non-linear relationship. The nonlinear relationship analyzed by these authors is between the mean absolute deviation and the market return, and their existence would be evidence of the herd effect. Originally, it is the Cross-Section Absolute Deviation of Returns (CSAD) variable.

The proposal is that the initial definition (in the formation) of stock weights in the portfolio should aim to maximize the fundamentalist score of the portfolio, that is, that the weights value more the stocks that are undervalued by the fundamentalist criteria. After the initial optimization of the portfolio weights, the proposed systematic optimization consists of periodically reviewing if the stock weights in the portfolio are adequate and do not need to be changed. In order to decide whether or not stock weights were changed in the portfolio, a joint analysis of some variables has been done. This joint analysis of indicators implies two possible scenarios which will lead to different decisions.

In the first scenario, the positive return of the stock in conjunction with the negative CSAD-Rm^2 correlation demonstrates the late performance of noise traders, which makes the stock overvalued (fundamentalist score decreases) and therefore changes the expectation of

[1] Professor of Finance at Federal University of Paraíba (UFPB) and PhD in Finance at UFPE
[2] Professor of Finance at Federal University of Pernambuco (UFPE) and PhD in Finance at PUC-Rio
[3] Professor of Finance at Federal University of Pernambuco (UFPE) and PhD in Finance at COPPEAD - UFRJ
[4] Professor of Finance at Federal Institute of Paraíba (IFPB) and PhD in Finance at UFPE

return (Beta changing). As the expectation is of reversing returns (since the arbitrators will check the movement and withdraw their positions), it is time to reduce the weight of these stocks in the portfolio.

In the second scenario, the negative return of the stock in conjunction with the CSAD-Rm^2 correlation demonstrates the late performance of the noise traders in withdrawing their positions, undervaluing the stock (increasing fundamentalist scores). As the expectation is of reversal of return (arbitrators will return to position in this portfolio), it is the moment to increase the weight of these stocks.

The results obtained in descriptive terms showed that the recommended portfolio would have a better return if applied the optimization by Mean-Variance only in the formation period, corroborating with the fact that it is an essentially fundamentalist portfolio. However, the hypothesis tests showed that the differences between the models were not significant, making the recommended portfolio when equally weighted more efficient than the other methods.

In the case of losing portfolios, in terms of descriptive results of the return, the systematic proposal would be the best. This is logical from the moment that the systematic proposal maximizes, among the losers, the weight of those more undervalued in relation to its fair price. However, the statistical tests were not significant, and it was also concluded that the equally weighted portfolio is the best because it reduces transaction costs.

In the case of the winning portfolio, the systematic proposal both in the formation period and periodically showed a very positive return and above the other optimization methods. These results were confirmed by the Friedman's Test, being insignificant only the relationship between the proposed systematic in the formation period and periodically. As the systematic proposal applied only in the formation period tends to have lower transaction costs, this method was initially chosen.

The results obtained corroborate with Bohm and Wenzelburger (2005), which demonstrated that rational investors' portfolios in the mean-variance plan are considered efficient ex-ante (portfolio formation) when compared to noise traders. However, the authors have shown that there is no direct relationship between their ex-ante efficiency and their ex-post empirical performance (as measured by the Sharpe Index), and that noise traders may have better ex-post performance than investors who base the choice of their portfolio in the efficient mean-variance plan.

The fact that the difference between the method of optimization by the Mean-Variance and the systematic proposal was not significant in the recommended and losing portfolios corroborates with the result of Das et. al. (2010), because they have proven that efficient sub-portfolio combinations in the mean-variance plan result in an efficient aggregate portfolio and that, since the approach to mental accounting is mathematically equivalent to that of the mean-variance, mental accounting-based subportfolios are also efficient in the mean-variance plane. In other words, the method proposed here is as efficient as Mean-Variance, and it is not possible to say which method is the best.

Keywords: Portfolio optimization; Behavioral Finance; Fundamentalist analysis.

1. INTRODUCTION

In the capital market and in finance theory, the fundamental assumptions are the rationality of investors and the maximization of expected utility. Thus, the expectation is that investors have rational behavior as drivers of investment decisions in order to obtain higher returns at lower risks.

These assumptions are already present in the 1950s, when one of the most important theories of finance was published, Harry Markowitz's seminal work (1952), entitled "Portfolio Selection", in which the author formulates basically two stages for portfolio selection investments.

The first stage corresponds to selection the assets that will compose the investment portfolio, through observation and analysis of the expected future performance of these investments. The second stage focuses on optimizing portfolio performance by choosing the weight of each asset in the portfolio. It is clarified that Markowitz (1952) focuses on the second stage, developing a methodology to optimize the portfolio for what is expected of a rational investment: the greater return to a certain level of risk or the lower risk for a given level of return.

One of the assumptions for Portfolio Selection Theory takes place in practice was the perfect functioning of the market, which the author himself considers as a limitation of his work, in order of imperfections that may invalidate the investor's rationality hypothesis (MARKOWITZ, 1952). In a progressive analysis of this perspective, Eugene Fama (1970) developed the Efficient Market Hypothesis (EMH).

According to Fama (1970), in markets with a certain degree of inefficiency, classified in EMH as weak or semi-strong, prices do not reflect all available information, generating the possibility of obtaining returns above average market, also called abnormal return, then emerging the concept of market anomaly.

One concept of market anomaly that can be cited is that given by Camargos and Barbosa (2003, p. 49), according to which anomaly is "the existence of regular patterns of asset´s returns behavior that do not fit into any of the theories about market efficiency ". These patterns being explained by the fact that a significant part of the investors are subject to the same information. Thus, although random, irrational behaviors are identified in the market, which constitute evidence unfavorable to EMH.

The seminal work that aimed to identify deviations of investors' behavior - which breaks the assumption of rationality - was that done by Kahneman and Tversky (1979). The authors have developed the Prospect Theory, according to which people are unable to be totally rational when they placed under uncertainty and rely on heuristic shortcuts or generic principles, causing market anomalies (unexpected returns behaviors when rationality of markets).

The occurrence of market anomalies brings the possibility of obtaining extraordinary returns from variables not considered by the traditional models used in the finance area for asset pricing, usually limited to a certain period of time and incorporating only the variations of the asset in relation to the market.

Thus, it can be assumed that historical values of stock returns can be used to predict future gains. If this forecast indicates a tendency to maintain the past returns of the asset, it is the Momentum Effect, as evidenced by Jegadeesh and Titman (1993) in the North American market. On the other hand, when a reversal of the past results occurs, the effect is known as Overreaction, expecting the behavior of the stock returns in the opposite direction, according to DeBondt and Thaler (1985).

And why do these changes in asset prices characterize themselves as behavioral effects? We can begin to explain precisely the concept of Momentum, in Damodaran's (2006) denomination, since favorable results to companies boost considerable number of investors to buy their stocks, generating upward movements in prices (Momentum Effect) until the market verifies that the asset has a market price above the fair, being the moment in which the quotations return to their fundamentals, reverting their results (Overreaction).

This adjustment of prices to fundamentals usually occurs in the longer term, as shares currently undervalued (but with potential growth opportunities) tend to reverse their results usually in a minimum period of three years and, more significantly, in five years.

Starting from Kahneman and Tversky's Theory (1979), some of the irrational behaviors of investors can be identified. For example, Ferreira (2008) explains overconfidence, a behavioral effect characterized by the belief that everything will happen as expected. Another behavioral deviation detailed by Ferreira (2008) is the herd behavior, in which investors imitate the attitudes of others, be they rational or not, and follow the flow of buying or selling stocks that are happening.

This herd behavior constitutes one of the explanations for the occurrence of the Momentum effect or maintenance of the returns, since it is expected that the stock values follow a trend in certain periods, it is admitted a violation of the assumptions of market efficiency even in its weak form, as historical price data become predictive of future returns (FAMA, 1970).

Herd behavior has been documented in the very short term (daily and weekly) by Kudryavtsev, Cohen and Hon Snir (2012), who conceptualize this effect as a result of the imitation of other investors. Chang, Cheng and Khorana (2000) modeled the herd behavior, being this effect measured from the standard deviation of the returns in relation to the market average in a non-linear relationship.

If the choice of Momentum effect as a strategy for selecting which assets to invest corresponds to the "first stage" of Markowitz Portfolio Selection Theory (1952), strategy and theory are fully compliant, because it is expected that winning stocks continue to have positive returns in the short term, and the "second stage" process - of optimizing the portfolio through the determination of weights - enhances the Momentum effect by giving greater weight to the stocks that bring to the portfolio result a greater return at the same level of risk, or a lower risk while maintaining the same level of return.

When the Overreaction Effect is chosen as a strategy for the selection of assets in the "first stage", hoping that in the long term (from three to five years) loser stocks will bring higher returns than the winners in the same period. The "second stage" of the Theory of Portfolio Selection creates a conflict with this choice of investment strategy, since it will give greater weight to the stocks that have given a greater return in the past to the same level of risk, and Overreaction expects a better performance of the assets with the lowest returns in the formation period.

Results such as those found by DeBondt and Thaler (1985) and Chopra, Lakonishock and Ritter (1992), which prove the existence of the Overreaction effect, lead to a contradictory thinking when analyze Markowitz's asset selection model (1952). This happens because assets selected for the portfolio are those with the best risk / return ratio, seeking the highest return to the lowest possible risk in the formation period (first year).

This better relationship is the contradictory question according to Overreaction, because by following this strategy, we look for stocks with poor performance in the formation period, with the expectation of reversion of returns in the long term. However, when one considers the Markowitz (1952) model for the definition of stock´s weights in the portfolio, when seeking the point of minimum variance maintaining the same level of return, the stocks with the largest weight in the portfolio will be those that have a larger return and a smaller variance, going against the idea of investing in Overreaction.

Given the possibility of explaining the occurrence of Momentum and Overreaction effects through the herd effect and the overconfidence, as did Statman et. al. (2006) and Jegadeesh and Titman (2011), it is in this "best relation between risk and return in the training period" that is found the core of this research.

Considering that the application of Markowitz Portfolio Selection Theory (1952) may bring contradictions when used in conjunction with investment strategies based on market anomalies caused by the herd effect and overconfidence (behavioral deviations), then the second stage of Portfolio Selection Theory should be used with a different systematic, which considers these behavioral effects.

Therefore, the research question is: Does a systematic optimization of portfolios that accompany the dynamics of the herd effect and the overconfidence in the Brazilian Capital Market can increase the return on investment in relation to Markowitz's (1952) method of portfolio selection?

In this way, the general objective of this research is to model a systematic optimization of portfolios that accompanies the dynamism of the herd effect and the overconfidence, if these are present in the Brazilian capital market.

2. LITERATURE REVIEW

The first section of this Literature Review discusses the Classical Finance Theories. The second section discusses the Behavioral Finance Theories, which are opposed to market efficiency and assumptions of investor´s rationality, and how the Herd Behavior and Momentum Effects can be explained by these theories. Finally, in the third section, the studies already carried out about behavioral portfolios Selection models, that have been systematized by researchers who have tried to adapt the model proposed by Markowitz (1952) over the years. This constitute as a link between the literature review and the methodology, in order to identify the novelty of this research in the idea that it has not yet been developed systematic optimization of portfolios that seeks to maximize the expected return by observing behavioral deviations in prices.

2.1 Classical Finance and the Portfolio Selection Theory

The study of Classical Finance begins in 1952 when Harry Markowitz's seminal work (1952) was published, entitled "Portfolio Selection", in which the author formulates basically two stages for selection of investment portfolios.

The first stage corresponds to the selection of assets that will compose the portfolio, through observation and analysis of expected future performance of them. The second stage focuses on optimizing portfolio performance by choosing the weight of each asset in the portfolio. It is worth noting that Markowitz (1952) focuses on the second stage, developing a methodology to optimize the portfolio for what is expected of a rational investment: the greater return to a certain level of risk or the lower risk for a given level of return.

The portfolio´s return, as shown by the cited author, is calculated as the weighted average of individual asset´s returns, where the weight of each asset in the portfolio is arbitrated by the investor (equation 1).

$$R_P = \sum_{i=1}^{n} R_i \times W_i \qquad (1)$$

Where: R_P is the return of portfolio *P*; R_i is the return of each asset *i*; and W_i is the weight of each asset *i* in the portfolio *P*.

Moreover, portfolio risk is measured by the portfolio´s variance, as established by the author, and involves the weights of assets, plus the contribution of each of them to the portfolio risk, measured by the variance, and the covariance between each pair of assets component of the portfolio, as expressed in equation 2 below.

$$VAR_P = \sum_{i=1}^{n} W_i \times (\sum_{j=1}^{n} W_j \times \sigma_{ij}) \qquad (2)$$

Where: VAR_P is the variance of portfolio *P*; W_i is the weight of each asset *i* in the portfolio *P*; W_j is the weight of each asset *j* in the portfolio *P*; and σ_{ij} represents the covariance between the pair of assets *i* and *j*, if *i* is different from *j*; or represents the variance of asset *i*, if *i* equals *j*.

On the geometric plane, Markowitz (1952) demonstrated the efficient frontier concept for a portfolio with four assets from a three-dimensional plane, generalizing it to a portfolio with *n* assets. In order to define the efficient frontier of a portfolio, it is necessary to estimate its expected return, called *E(Ri)*, as well as its expected risk. According to Markowitz (1952), the expected return on a portfolio is measured by the weighted average of the expected returns of each asset that makes up this portfolio, while the expected risk is measured by the variance of the portfolio, which is calculated by the variance sum of each asset with covariance between assets, at par.

Without the pretension of exhausting all possible estimators of expected return, the literature review is followed by the description of an expected return on an asset - which is based on the risk premium of the asset - and is well known in the market: the Capital Asset Pricing Model (CAPM), which was developed by Sharpe (1964), Lintner (1965) and Mossin (1966), and can be calculated as:

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f] \qquad (3)$$

Where: *E(Ri)* is the expected return on asset *i*; *Rf* is the return of risk-free asset; *βi (beta)* is the relationship between the return´s variation of asset *i* and that of the market portfolio; and *E(Rm)* is the expected return of market portfolio.

In this way, it is possible to estimate the expected return of an asset as a sum between the return of risk-free asset (*Rf*) and the premium for risk of the asset *i*, which consists of the difference between the expected market return (*Rm*) and *Rf*, adjusted by the asset risk factor (beta). Having then developed a concept of estimating the expected return on assets through the CAPM, the data generated by CAPM can be used to calculate the expected return on the portfolio, and the historical asset return data is used to calculate the variance of each one and the standard deviation of the portfolio, which is the risk measure.

Markowitz (1952) found that the risk of a portfolio was less than the sum of standard deviations of individual components in that portfolio, and thus evidenced the benefits of diversifying assets. Sharpe (1964) explained the benefit of diversification from the division of risk into two parts: (1) systematic or non-diversifiable risk; and (2) non-systematic or diversifiable risk.

The β factor represents the systematic risk of the asset, which is the risk inherent in market variations, and it is not possible to control it. Non-systematic risk is one that is linked to the specific characteristics of the company and can be reduced by diversifying the assets that make up the portfolio.

Both Sharpe (1964) and Markowitz (1952) affirmed in their studies the fact that for their models worked, the perfect functioning of market was assumed, which Markowitz (1952) considers as a limitation of his work, given that there are imperfections in the market that may invalidate the investor's rationality hypothesis. In a progressive analysis of this perspective, Eugene Fama developed the Efficient Market Hypothesis - EMH (1970).

According to Fama (1970), in markets with a certain degree of inefficiency, ranked in the HME as weak or semi-strong, prices do not reflect all available information, generating the possibility of achieving above-average returns, also called abnormal returns, creating the concept of market anomaly.

One concept of market anomaly that can be cited is given by Camargos and Barbosa (2003, p. 49), according to which anomaly is "the existence of regular patterns of asset´s returns behavior that do not fit into any of the theories about market efficiency ". These patterns being explained by the fact that a significant part of the investors are subject to the same information. The occurrence of market anomalies allows opportunities to obtain extraordinary returns from variables not considered by the models traditionally used in the finance area for asset pricing.

2.2 Herd Behavior and its Behavioral Bases

Kahneman and Tversky's (1979) seminal work about Prospect Theory contrasts with the investor's rationality hypothesis, providing a possible explanation for the inefficiency levels of capital markets and their anomalies. From Kahneman and Tversky's Theory (1979), some of the irrational behavior of investors can be identified. For example, according to Ferreira (2008), one of the main behaviors identified is the overconfidence, a behavioral effect characterized by self-confidence and the belief that everything will happen as expected. Another behavioral deviation detailed by Ferreira (2008) is the herd effect, in which investors imitate the attitudes of others, be they rational or not, and follow the flow of buying or selling assets.

Taking into account the results of classical authors, the Momentum strategy is based on the short-term, consisting of the investor´s expectations for which stocks that had a significant appreciation in the recent past are likely to continue growing in the future. One of the classic studies about Momentum in the North American market was done for Jegadeesh and Titman (1993), which evidenced the maintenance of returns in the three, six, nine and twelve months following the portfolio´s formation.

Introducing evidence that the momentum effect is not dealt with a skewed results inherent to the period documented by Jegadeesh and Titman (1993), which was from 1965 to 1989, the same authors argue that the effect continued in the 1990s (JEGADEESH AND TITMAN, 2001). It is now analyzed with more emphasis on the behavioral aspects, possibly explaining the maintenance of returns in the short term. In a complementary way, they confirm the overreaction effect in the period from thirteen to sixty months, corroborating with the results of authors such as Debondt and Thaler (1985) and Chopra, Lakonishok and Ritter (1992).

The classic authors of this strategy return to the subject by emphasizing the current relevance of Momentum strategy and that it constitutes one of the strongest evidence against market efficiency (JEGADEESH and TITMAN, 2011). Seeking to demonstrate explanatory factors of this market anomaly, they make a survey of the main studies already done demonstrating the theoretical and empirical antecedents of the Momentum effect, being the behavioral factor one of the main causes for gains maintenance.

This behavior of following impulsively the flow of the negotiations has been documented, in the very short term (daily and weekly), as a herd behavior or herd effect. Kudryavtsev, Cohen and Hon Snir (2012) conceptualize this effect as arising from the act of imitating other investors to the detriment of one's own beliefs and information about the assets.

Herd behavior had previously been modeled by Chang, Cheng, and Khorana (2000), being the effect measured from the standard deviation of returns in relation to the market average in a non-linear relationship. The authors clarify the linearity assumption of traditional asset pricing models, based on the Efficiency Market Hypothesis, according to which price would reflect all available information and there would be no anomalies such as the herd effect. The nonlinear relationship analyzed by these authors is between the mean absolute deviation and the market return, and their existence would be evidence of the herd effect. Originally, it is the Cross-Section Absolute Deviation of Returns (CSAD) variable, suggesting that market participants are predominantly based on macroeconomic information rather than more specific information and thus provoke non-linear movements in the returns deviations.

2.3 Behavioral Finance and Portfolios Selection

The abnormal returns (far above or well below the market average) can not be explained by a situation of equilibrium, indicating a market inefficiency that can be caused by several factors. Among them, the main thing is the breakdown of maximizing expected utility assumption by the investor, indicating not be fully rational because of misbehavior, as studied by Kahneman and Tversky (1979) in the development of Prospect Theory.

On this basis, Shefrin and Statman (1994) developed the Behavioral Asset Pricing Model (BAPM), considering that in the financial market there are not only rational investors, such as those mentioned by Markowitz (1952) and Sharpe (1964), but also the noise traders, typically irrational investors.

These authors report that there is a single price targeting property, which is the amount of information needed to generate changes in the distribution of returns. Noise traders include in the market a second targeting of prices, their affect, making decisions that are not based on fundamentalist information of the firms and neither on historical data of prices, but on emotions, contributing to the market inefficiency and occurrence of anomalies (SHEFRIN AND STATMAN, 1994, p. 324). Thus, they calculate the expected return of an asset by CAPM plus an expectation of abnormal return.

Shefrin and Statman (1994) point out that if markets are not efficient, investors are then calculating the betas of the assets against an inefficient market portfolio, which can be represented by βi and means the true beta of asset *i*. If there is an inefficient market portfolio, it is possible to set up an efficient market portfolio, which will be the benchmark portfolio. So, emerges the concept of β_M, which means the relationship between the efficient market portfolio and the inefficient (real) market portfolio. If β_M equals 1, it means that the real market portfolio is efficient. Therefore, when the ratio (βi / β_M) is made, the relation with the inefficient portfolio is removed, "correcting the Beta", according to the authors themselves.

The above-mentioned authors return to research in the area of behavioral finance a few years later (SHEFRIN and STATMAN, 2000), developing the Behavioral Portfolio Theory (BPT), which is based on the mental accounting framework developed as the basis of Prospect Theory.

According to Kahneman and Tversky (1979), mental accounting is the process by which individuals visualize their investments in separate accounts, according to the objectives defined for each one, leading to the inefficiency and the irrationality of the investment decision process. In this way, Shefrin and Statman (2000) develop the BPT considering first that there is a single mental account, the BPT-SA (Single Account), and then considering that the investors divide their investments into two mental accounts: the one that seeks to avoid poverty and one that seeks the generation of wealth, then creating the BPT-MA (Multiple Account).

These same authors report the differences between investors who are based on the mean-variance, the CAPM and the BPT. While investors based on the mean variance look for assets with this better ratio, those based on the CAPM have portfolios formed by risk free assets and the market portfolio. Finally, those based on the BPT consider the hope of wealth, levels of aspiration and probabilities of occurring, while investing in lottery tickets (SHEFRIN AND STATMAN, 2000, p. 128).

While in Markowitz (1952) the weights are defined in order to maximize the expected return for a given level of risk, in the BPT-SA the efficient boundary is obtained by maximizing the expected return to a given probability level that the expected return is lower than the aspirated level "A".

More recently, Das et. al. (2010) integrated the portfolio theories of Markowitz (1952) and Shefrin and Statman (2000) into a new mental accounting framework, which demonstrates a mathematical equivalence between mean-variance and BPT through the concept of mental accounting. According to the authors, the objective of the research is to find the optimal portfolio having as risk perception the probability of not achieving specific objectives defined in each mental account.

Das et. al. (2010, p.2) point out that each mental account has a specific efficient frontier, based on its expectation of return and the probability of not reaching the desired level, and that in this situation investors may wish to take risks, whereas in the approach of average-variance investors are always averse to risk.

When verifying that the mean-variance and BPT approaches are mathematically equivalent, Das et. al. (2010) show that the subportfolios of each mental account have an implicit coefficient of risk aversion, and that investors are more apt to report their financial goals than their level of risk aversion, and more so when they are divided into mental accounts.

However, the results of the research by Das et. al. (2010) that most apply to this research consist in proving that combinations of efficient subportfolios in the mean-variance plane results in an efficient aggregate portfolio. As the mental accounting approach is mathematically equivalent to that of mean-variance, mental accountability-based subportfolios are also efficient in the mean-variance plane.

3. METHODOLOGY

In order to contextualize how the theoretical bases developed contributed to the formulation of the research hypotheses, it is recalled here the general objective of this research, which is to seek a model of a systematic optimization of portfolios that accompanies the dynamism of the herd effect and the overconfidence, if these are present in the Brazilian capital market.

For the development of these hypotheses, when examining the main theoretical references in the areas of Classic Finance and Behavioral Finance, a possible relationship between the market situation in equilibrium proposed by Sharpe (1964) during the formulation of the CAPM, and the Behavioral Asset Pricing Model (BAPM), proposed by Shefrin and Statman (1994). The joint study of both capital asset pricing theories (based on Classical Finance and Behavioral Finance) and the Momentum and Overreaction effects brought the inclination to formulate a simulation about how capital assets move in the mean-variance.

Noting the theoretical bases analyzed and presented in the literature review, it is reflected that possibly the assets move their positions in the mean-variance plan, and that because the markets are not in balance due to the breakdown of the rationality assumption of investor, they may not move in order to transform the efficient boundary of a curve into a linear relationship, according to Sharpe (1964).

In this way, it is reflected that - possibly - assets move their positions over time in the mean variance plane in the sense of constructing this equilibrium proposed by Sharpe (1964), but because of the performance of the noise traders, the position of each asset is alternating between above the capital market line and below the same line, passing through the efficient frontier at some points during this move. Because of this move that goes from a point above the capital market line to a point below, the Momentum and Overreaction effects happen constantly.

Because of these movements, the Momentum and Overreaction effects are assumed to occur on a regular basis, with a possible explanation that the inversion of betas between the winning assets and the losing assets occurs due to the performance of the noise traders in the market. This assumption corroborates the studies of Black, Jensen and Scholes (1972) and DeBondt and Thaler (1985), which affirms that the betas of the assets vary over time.

Therefore, based on this contextualization between the research objective and the theoretical bases studied, it was possible to formulate the hypotheses of this study, in order to make clear what is being tested. In this way, there are three hypotheses to be tested.

The first hypothesis is that there is a significant difference and a negative correlation between the stock returns traded on BM&FBOVESPA (the Brazilian Stock Exchange) in the portfolio formation periods (twelve months) and the return verified in the sixty months thereafter, indicating the existence of Overreaction in the Brazilian capital market between 1995 and 2015. In order to test this hypothesis, the methodology used by Debondt and Thaler (1985) and Chopra, Lakonishok and Ritter (1992) was replicated for Brazilian capital market.

Not being rejected the Overreaction hypothesis in the Brazilian capital market, it means that the winning stocks become losers in the long term (from three to five years), and vice versa. As the study is developed sequentially, in which one stage depends on the other, the non-rejection of the Overreaction hypothesis reinforces the second part of this study, which seeks justifications for the occurrence of this anomaly.

Since a part of the stock return can be caused by undervaluation of this stock, while another part may be caused by the deviant behavior of noise traders, it is necessary to choose models that predict these returns. To identify the return caused by this undervaluation, the Five-Factor Model of Fama and French (2015) was chosen. The difference between the real and predicted returns was called "abnormal return", and then analyzed if this abnormal return is caused by behavioral deviations from noise traders using the methodology of Chang, Cheng and Khorana (2000).

Thus, the second hypothesis of this research is defined: There is a significant difference between the real returns of the stocks in the period from 1995 to 2015 and those predicted by the Five-Factor Model of Fama and French (2015), being this difference explained by the performance of noise traders, according to the method of Chang, Cheng and Khorana (2000).

Having seen this, we proceed to the construction of the hypothesis that carries the central part of this research, which is to develop a systematic of optimization portfolios that accompanies the dynamism of these behavioral effects in the capital markets, enhancing the returns of investment portfolios. This systematic proposal is based on non-rejection of Hypotheses I and II, thus seeking a method that gives greater weight in the portfolio to assets that are undervalued due to the performance of noise traders in the market, that is, identifying the point at which the expectation of return will reverse and thus maximizing the expectation of portfolio return.

After the optimization systematics formulation, it will be verified if the returns of the analyzed portfolios are significantly higher by the optimization proposal or if the traditional optimization model proposed by Markowitz (1952) is a better estimator. Based on this concept, the third hypothesis is defined: There is a significant difference between portfolio returns optimized according to Markowitz's (1952) method of portfolio selection and the systematic proposal, which considers the dynamism of behavioral effects in the Brazilian capital market, in the period from 1995 to 2015.

The non-rejection of the three hypotheses proposed in this research creates a coherent logic of the Overreaction effect in the Brazilian capital market, being part of this anomaly explained by the behavioral deviations of the noise traders, and so it is possible to use this knowledge to maximize the return of investment portfolios through a systematic optimization of portfolios that accompany this dynamism.

This paper concentrates only on the detail of third hypothesis, initially reflecting the results obtained for the previous hypotheses. It is considered that most part of this paper is presented in the following sections, with a view to being a methodological proposal.

3.1 Database Organization

To test the hypotheses proposed, an empirical application was made, and for this the universe considered is the database of the stocks traded on the BM&FBOVESPA (Brazilian Stock Exchange), from 1995 to 2015, accessed through the Economatica system.

From the defined universe, the sample's selection that composes the analysis was defined by the following exclusions, applied to each analyzed period: (1) The type of asset analyzed are stocks of non-banking institutions (excluding ADRs, Indexes, stocks of the Sector denominated "Finance and Insurance" and other types of assets); (2) the stocks must have active trading status (therefore, those canceled are excluded); and (3) for the data consistency necessary for statistical analysis in terms of liquidity, only those with at least monthly negotiation in training were considered.

The entire database was adjusted for inflation until the last available date, which is September 21, 2016, and further adjusted for earnings, including dividends. These adjustments are made automatically in the Economatica system.

Once the sample has been defined, we start with the definition of variables necessary for the analysis. Since all assumptions focus on past returns as predictive of future returns, the main variable is the return of stocks. The "return" variable is sufficient for the analysis of hypothesis I, but hypothesis II needs six more variables, necessary for the construction of the Five-Factors Model of Fama and French (2015).

In order to subsidize the construction of the systematic portfolio optimization proposal (analysis of Hypothesis III), two other variables had to be collected: one of them being Beta (β), and the other being the volume traded, both in terms of quantity stocks traded and volume of money, in order to monitor the liquidity of the portfolios that will be formed to analyze and to be used as a decision variable. The entire database described here was assembled in spreadsheets of Microsoft Excel 2010, being a spreadsheet for each period of analysis.

In order to test the proposed hypothesis I, winning and losing portfolios were created in the Brazilian capital market - composed of the 15 stocks with the highest returns in the year of formation, and the 15 stocks with the lowest returns in the year of formation, respectively - and analyzed their returns over the next five years.

The selected amount of 15 stocks per portfolio corroborates the studies of Brito (1989) and Ceretta and Costa Jr (2006) about the ideal quantity of assets that make a diversified portfolio in the Brazilian capital market (balancing non-systematic risk reduction and transaction costs).

In order to have a greater amount of data and generate a more significant sample, the winning and losing portfolios were created every year for 16 years (from 1995, when the Real

Plan was implemented, until the year 2010), and analyzed in the following five years (1996 - 2000 for the first portfolio and 2011 - 2015 for the last portfolio).

Thus, the total number of portfolios to be analyzed in Hypothesis I is 32, with 16 of them winning in their training period and 16 losers in the same period.

3.2 Process optimization of control portfolios

Once analyzed Hypotheses I and II, sufficient information was obtained to propose a new systematic optimization of the portfolios by defining the periodicity of rebalancing stock's weights in the portfolio and the indicators that would be followed for the decision to change weights or not.

However, before demonstrating the methodological procedures of systematic optimization developed here, we present in this section the optimization processes of control portfolios, that is, the portfolios that were used as a way of comparison, in order to evaluate if the systematics created brings or no better performance in terms of long-term return than the traditional optimization methods.

It is necessary to clarify the return and risk estimators of the portfolios, independent of the optimization process used (if traditional or with the systematic proposal). Here we do not have the intention to modify the risk (variance of the portfolio) and return (weighted average of the individual asset returns) estimators traditionally used by Markowitz (1952), because it would make the search go the other way, being the focus here on alteration of the optimization process (techniques to define stock weights in the portfolio).

For this stage of analysis, the winning and losing portfolios are composed respectively by the 15 stocks with the highest returns in the training period and the 15 stocks with the lowest returns in the training period selected. As the database allows 16 iterations, 32 portfolios will be analyzed. In addition, the portfolio created for the systematic proposal application also went through the traditional processes of optimization. Therefore, 48 portfolios are counted, being 16 winners, 16 losers and 16 created as recommended portfolio for the application of the systematics developed. As these 48 portfolios went through three different optimization processes, there were $48 \times 3 = 144$ portfolios with different methods being monitored.

The formed portfolios underwent two traditional process optimization, to see which ones generate the highest return performance. There are two traditional ways to weight the selected portfolio: (1) equally weighted portfolio, in which the weights are equal for all assets (1/N); and (2) the weighted portfolio by mean-variance (Markowitz, 1952), in order to minimize the risk to maintain the same level of return. Let's look at each one of them. About equally weighted portfolio, the weight of each asset in the portfolio is determined by the equal participation of each stock in the portfolio.

On the mean-variance portfolio, Markowitz (1952) determined a risk minimization function by maintaining the same level of return, or maximizing the return to a given level of risk. Thus, given the same level of risk calculated for the equally weighted portfolio, the weights of assets in the mean-variance portfolio have been determined by:

$$W_{i-MV} = f(Max_{F_{t-1}} R_P) \quad (4)$$

Where: $W_{i\text{-}MV}$ is the weight of each asset i in the portfolio measured by the mean-variance method; f(MaxR_P) is the function of maximization the return's portfolio (R_P); and F_{t-1} is the formation period, keeping all other variables constant.

It is worth considering that, for the two cases of weighting mentioned above and for what will be proposed, there are two restriction conditions: (1) $\sum_{i=1}^{N} W_i = 1$, which means that the sum of the stock weights in the portfolio should be equal to 100%; and (2) $W_i \geq 0$, which means that no stock weight can be negative. In addition, in the case of optimization by the mean-variance and in the systematic proposal that will be elaborated, a third restriction consists to maintain or reduce the risk in relation to the equally weighted portfolio. All optimizations have been calculated in Solver, a Microsoft Excel 2010 add-in, as did Gonçalves Júnior, Pamplona and Montevechi (2002).

3.3 Systematic Proposal for Portfolio Optimization

Once Hypotheses I and II have been analyzed, information was obtained to feed the necessary data to the new portfolio optimization systematic, through the periodic analysis of these indicators for the decision to rebalance stock weights in the portfolio. These indicators and the expectation of results for each of them (according to the literature) are presented in Table 1.

Table 1 - Indicators for the Creation of Portfolio Optimization Systematics

Variable	Category	Participation	Expectancy
Return Observed of the stock (Ri)	Dependent Variable	Monitoring and Scoring	Dependent Variable
Market Value (MV)	Fundamentalist	Scoring	The lower MV, higher is the return's expectation
Book-to-Market (BM)	Fundamentalist	Scoring	The higher BM, higher is the return's expectation
Operational Profitability *(OP)*	Fundamentalist	Scoring	The higher OP, higher is the return's expectation
Growth rates of total assets *(INV)*	Fundamentalist	Scoring	The lower INV, higher is the return's expectation
Stock Beta (β)	Relationship with the Market	Monitoring	If varies, it indicates change of return expectation
Volume Negotiated in Quantity (*Volq*)	Liquidity	Monitoring	Analyze in conjunction with *Correl_CSAD*
Volume Negotiated in Money (*VolM*)	Liquidity	Monitoring	Analyze in conjunction with *Correl_CSAD*
Correlation between *CSAD* and Rm^2 (*Correl_CSAD)*	Behavior of noise traders	Monitoring	If it is negative and significant, it indicates the performance of noise traders.

Source: Own elaboration

The proposal of the monitoring system consists in the definition of a stock scoring criterion to select the 15 stocks that will be part of the recommended portfolio to be monitored and then optimize it. This criterion of scoring is divided into two parts, the first of which refers to the creation of a measure that indicates a fundamentalist advantage of investing in that stock, while the second measure is to evaluate the whole context together, also considering the behavioral variables, according to scenarios described later.

First, in relation to the score that will indicate a fundamentalist advantage of investing in that stock, the four indicators considered were those present in the Five-Factor Model of Fama and French (2015), according to the expectations predicted by the authors. Thus, while high Book-to-Market (BM) and high operating profitability of Equity (OP) indicate undervalued stocks, a low Market Value (MV) and lower growth rates of total assets (INV) reinforce this undervaluation.

Thus, the first scoring criterion (to verify the level of undervaluation by fundamentalist criteria) was to apply the "RANK" function of Microsoft Excel 2010 to these indicators, in order that the most undervalued stock would have the highest number of points (last place in the classification) and the most overvalued stock had the lowest number of points. Therefore, for the BM and OP indicators, the function was used by applying the ascending order, while the descending order was applied to the MV and INV indicators.

The final number of scores for each stock regarding their level of undervaluation consisted of the sum of these scorings. Therefore, the stock with the highest score would be considered the most undervalued relative.

$$S_{i,t} = \sum(S_{MV} + S_{BM} + S_{OP} + S_{INV}) \tag{5}$$

Where: $S_{i,t}$ is the fundamentalist score of the stock *i* in the portfolio in period *t*; and S_{MV}, S_{BM}, S_{OP} and S_{INV} are the scores for each one of the four indicators.

The non-rejection of Hypothesis I of this research (the occurrence of the Overreaction effect) made this first criterion of scoring count with one more indicator: the observed return in the formation period. If this is negative, the non-rejection of Hypothesis I corroborates so that the highest score is given to the stocks with negative returns in the formation, since a reversion of these returns in the long term is expected. Thus, the final sum of points in this first part of the analysis had five indicators.

The proposed optimization system starts with the selection of the 15 stocks that have the highest score by the fundamentalist criteria presented here. As said in the previous section, in addition to this portfolio created from the systematic proposal, the winning and losing portfolios were also monitored, both the return of these portfolios and the score established herein. Table 2 shows a summary of the portfolios monitored and the optimization methods used. Since the portfolios (although with equal stocks, but with different optimization methods) are assembled in 16 different time iterations, the total number of portfolios being analyzed is 9 × 16 = 144 portfolios.

The proposal of a systematic optimization of stock weights in a portfolio does not only end with the choice of the stocks that will compose the portfolio, since it would be fulfilling only the first stage of Markowitz (1952).

Table 2 - Portfolios Monitored in the Result Analysis of Hypothesis III

Portfolio	Stock Selection	Optimization Method
P1	15 Winning stocks	Equally Weighted
P2	15 Winning stocks	Mean-Variance
P3	15 Winning stocks	Proposed Optimization Systematics
P4	15 Loser stocks	Equally Weighted
P5	15 Loser stocks	Mean-Variance
P6	15 Loser stocks	Proposed Optimization Systematics
P7	15 stocks with the highest fundamentalist score	Equally Weighted
P8	15 stocks with the highest fundamentalist score	Mean-Variance
P9	15 stocks with the highest fundamentalist score	Proposed Optimization Systematics

Source: Own elaboration

The proposal is that the initial definition (in the formation) of stock weights in the portfolio should aim to maximize the fundamentalist score of the portfolio, that is, that the weights value more the stocks that are undervalued by the fundamentalist criteria. Therefore, the weights have been calculated according to equation 6:

$$W_{i,t} = f(Max_{F_{t-1}} S_P) \quad (6)$$

Where: $W_{i,t}$ is the weight of each stock *i* in the portfolio in period *t*; f(MaxS_P) is the function of maximizing the fundamentalist Score of the portfolio *P* in the formation period Ft-1.

After the initial optimization of the portfolio weights, continues with the formulation of the proposed systematic optimization of portfolio, which consists of periodically reviewing if the stock weights in the portfolio are adequate and do not need to be changed. In order to decide whether or not stock weights were changed in the portfolio, a joint analysis of five measures was done, namely: (a) the observed return; (b) the score based on fundamentalist criteria; (c) the Beta of the stock; (d) the liquidity of the share; and (e) the correlation between CSAD and Rm^2. This joint analysis of indicators implies two possible scenarios which will lead to different decisions (Table 3).

Table 3 presents the scenarios that form the basis for the decision to reduce or increase stock weights in the portfolio. The periodicity with which these indicators were analyzed could only be defined after the study of Hypothesis I, which resulted in the existence of annual cycles. If the full scenario as described does not occur, the decision will consist of maintaining the current portfolio weights until the next round of decision.

The application of scenarios described above was done considering the execution of an IF Function in Microsoft Excel 2010, applying the two scenarios in each portfolio stock, generating binary variables as a result. In the case of the first scenario, if it occurs, it receives the result "zero", indicating a decrease in the weight of that share in the portfolio, and the value of "1" if the scenario does not materialize completely. In the case of the second scenario, its implementation will cause the stock receives the result "2", and the value of "1" if it is not consolidated.

Table 3 - Possible Scenarios in the Proposed Systematics

Context of Variables	Interpretation	Optimization Decision
1st scenario • Positive returns • Decreasing score • *Beta* of stock changing • Increasing liquidity • Correlation *CSAD*-Rm^2 negative	The positive return of the stock in conjunction with the negative CSAD-Rm^2 correlation demonstrates the late performance of the noise traders, which makes the stock overvalued (fundamentalist score decreases) and therefore changes the expectation of return (Beta changing)	As the expectation is of reversing returns (since the arbitrators will check the movement and withdraw their positions), it is time to reduce the weight of these stocks in the portfolio.
2nd scenario • Negative returns • Increasing score • *Beta* of stock changing • Increasing liquidity • Correlation *CSAD*-Rm^2 negative	The negative return of the stock in conjunction with the CSAD-Rm^2 correlation demonstrates the late performance of the noise traders in withdrawing their positions, undervaluing the stock (increasing fundamentalist scores)	As the expectation is of reversal of the return (arbitrators will return to position in this portfolio), it is the moment to increase the weight of these stocks.

Source: Own elaboration

In the end, as the scenarios differ, the sum of their scores yields only three results, "1" referring to the decision to decrease the weight of the share in the portfolio, "2" refers to weight maintenance and "3 "Refers to the share weight increase in that period (Table 4).

Table 4 - Possible Notes for Stocks in the Scenarios

Situation	Note in the 1st scenario	Note in the 2nd scenario	Final note
Decrease stock weight in the portfolio	0	1	1
Keep the stock weight in the portfolio	1	1	2
Increase the stock weight in the portfolio	1	2	3

Source: Own elaboration

In this way, and from the periodicity of analysis for rebalancing to be defined in the study of Hypotheses I and II, if all the stocks received a note "2" in the scenarios, the weights will not be modified in that period. If any one of them has received "1" or "3" notes, the optimization of weights will be done, seeking to maximize the portfolio's fundamentalist score, which is weighted by stock weights and scenario scores.

Thus, this development of stock weighting in the portfolio based on the Overreaction effect and on the identification of the noise trader's behavior seeks to identify the location changes of the stocks in the mean-variance plane, following the change of their betas and, consequently, their expectation of return, recalculating the weights in points defined in the period of time analysis, when the expectation of their return is reversed (from positive to negative and vice versa).

3.4 Procedures for analyzing the results of hypothesis

After performing the traditional optimization processes and with the new systematic proposal in all the portfolios, it is necessary to analyze the results obtained in terms of performance and the indicators monitored in each portfolio. Firstly, in a more descriptive approach, graphs and tables will be presented that will indicate the performance of each portfolio in terms of cumulative returns over the years of analysis. In order to clarify the methods that will be presented in the analysis of hypothesis results, it is recalled.

- H_0: There is not a significant difference between portfolio returns optimized according to Markowitz's (1952) method of portfolio selection and the systematic proposal, which considers the dynamism of behavioral effects in the Brazilian capital market, in the period from 1995 to 2015.
- H_1: There is a significant difference between portfolio returns optimized according to Markowitz's (1952) method of portfolio selection and the systematic proposal, which considers the dynamism of behavioral effects in the Brazilian capital market, in the period from 1995 to 2015.

Therefore, the expectation is that the optimization system developed will have a greater return than the traditional forms. After the presentation and discussion of the descriptive analyzes of the portfolios, we seek to reject or not to reject the hypothesis established by means the portfolios' performance. Initially, portfolio performance was measured individually by the Sharpe Index, as did Bohm and Welzenburger (2005) and Das et. al. (2010) in behavioral studies, since the total risk measure also considers non-diversified portfolios, while the Treynor Index is most effective in measuring the performance of well diversified portfolios.

As this research also includes not-so-diversified portfolios, there is a better application for the Sharpe Index. Thus, using the Sharpe Index to measure the performance of individual portfolios, it was possible to perform a comparison between these portfolios, classifying according to the optimization methods used and according to the periods of analysis.

So far, the techniques described can be used to compare the different portfolios, but it is necessary to statistically demonstrate the effectiveness of the proposed optimization system (which has two bases, optimization based on fundamental analysis and periodicity of weight rebalancing). For this, we used the differences-in-differences estimation, which consists in analyzing the differences between treatment and control groups before and after the intervention.

In order to use differences-in-differences estimation, the following steps were applied: (a) the three portfolios analyzed (the formed by the 15 stocks with the highest fundamentalist score, the winning portfolio and the losing portfolio) were optimized by the method of mean-variance (MARKOWITZ, 1952) and by the systematic optimization proposed using data from the formation period; (b) After defining the periodicity of rebalancing, these portfolios were rebalanced for each period provided by both the mean-variance method and the proposed optimization systematic; (c) The cumulative return of the portfolio was calculated if only the first definition of stock weights and the rebalancing were done, for the two methods chosen. This information is summarized in Table 5, below.

From the analysis of Table 5, it is expected that the cumulative (A-C) and (B-D) differences are positive, since it is expected that the systematic proposal will not reject the hypothesis that it yields a higher return than traditional optimization methods of portfolios. On the other hand, it is expected that the differences (A-B) and (C-D) are negative, since the periodic rebalancing of the portfolio according to the analysis of the indicators is expected to bring higher returns than to perform a single weighting decision at the beginning of the period. Therefore, it is expected that the difference of the difference [(B-D) - (A-C)] is positive, since it is expected that the systematic optimization proposal will yield a higher return than the mean-variance optimization, increased by the periodicity of rebalancing.

Table 5 - Estimation of Differences-in-Differences

Optimization Method	**If the optimization method were applied only once time**	**Periodically rebalanced stock weights**
Proposed Optimization Systematics	**A** Portfolio optimized by the systematic proposal only at the beginning of the analysis period	**B** Portfolio optimized by the systematic proposal periodically during the period of analysis
Mean-Variance (MARKOWITZ, 1952)	**C** Portfolio optimized by mean-variance only at the beginning of the analysis period	**D** Portfolio optimized by the mean-variance periodically during the period of analysis

Source: Own elaboration

4. ANALYSIS AND DISCUSSION OF RESULTS

This section begins with a summary of the results concerning Hypotheses I and II, since this paper focuses on the results of Hypothesis III, but cannot ignore the first two steps, since there is a linked relationship between them. From the analysis performed, we cannot reject the hypothesis of overreaction in Brazil in the long term (five years), when the loser stocks become winners and vice versa. This statement is based on the result of the Mann-Whitney U mean comparison test.

This analysis showed that, on average, the losing portfolio starts to gain from the winning portfolio from the 12th month analysis. Given these cycles presented by analysis, it was decided that the rebalancing frequency of portfolios during the construction of scenario III would be made every year, totaling then five tests for rebalancing: in the portfolio formation and at the end of the first four years of analysis.

The second part of results analysis, referring to Hypothesis II, identifies a coherence with the results expected by Fama and French (2015) in its five-factor model, generating a reversal in some of variables during the analysis period, which is characteristic of Overreaction. Already to explain the behavioral deviations of the investors, the CSAD has been calculated, and a significant result was found as a demonstrator of the presence of investor's behavior deviations in general in the Brazilian capital market. It was also possible to show that prices deviated from those expected by rational models in function to behavioral deviations.

Now detailing the results of Hypothesis III, the optimization methods were applied and the returns have been analyzed during 20 quarters or five years of analysis. The first result, concerning to the portfolios returns, is shown in Figure 1.

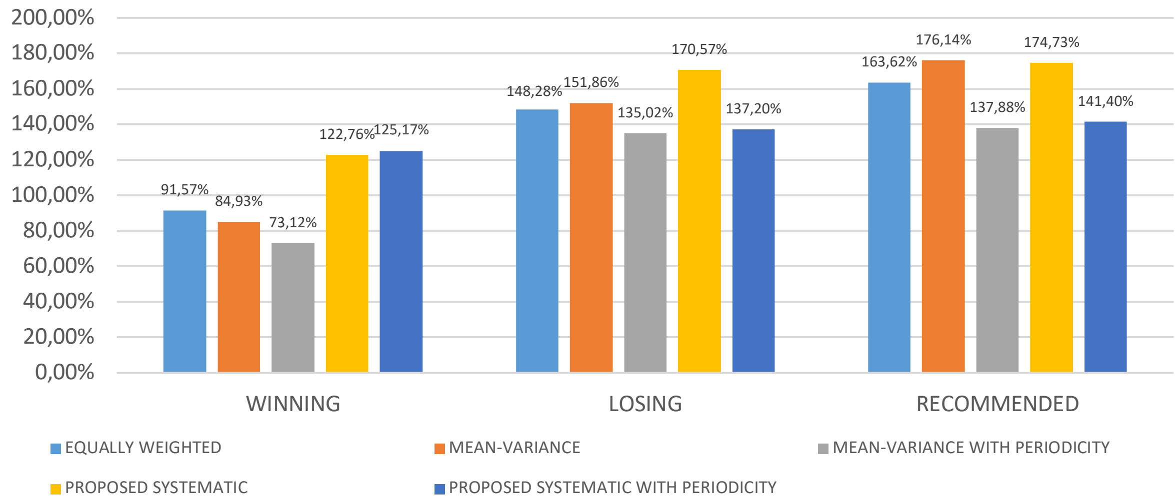

Figure 1 - Average Cumulative Return on Portfolios in the 20th Quarter of Analysis

Source: Own elaboration with the aid of Microsoft Excel 2010

Starting the descriptive analysis by the winning portfolio, it can be seen that the cumulative average return in 20 quarters when the stocks are equally weighted is 91.57%, and that the application of optimization by Mean-Variance (both only in formation and periodically) decreased the portfolio's profitability. However, when applied the systematic optimization proposal, the average return of the winning portfolio increased to 122.76% when applied only in formation and increased to 125.17% when applied annually.

Now analyzing the losing portfolio, it is clear that the fact of applying both the Mean-Variance optimization and the systematic proposal annually makes the return of the losing portfolio decrease. When these methods were applied only in portfolio formation, the return increased from 148.28% to 151.86% by the Mean-Variance method. Already applying the systematic proposal only in the portfolio formation, the return increased to 170.57%.

Looking now at the recommended portfolio, its return is better than the other portfolios (winner and loser), even when they are equally weighted. As in the losing portfolio, the periodic application of the two optimization methods analyzed made the cumulative average return in 20 quarters decrease. However, the application of the two methods only in the period of portfolio formation brought an increase of the returns, being the optimization by Mean-Variance a better performance than the systematic proposal.

Looking at the same data from another perspective (that of optimization methods), the recommended portfolio is better than the losing portfolio, and the losing portfolio is better than the winning portfolio, independent of the optimization methods. It is noticed that the application of both methods with periodicity in the losing and recommended portfolios reduced the return, while increasing the return of the winning portfolio.

This interpretation shows that the winner portfolio is one in which more is the herd effect, deviant behavior of investors studied here. Therefore, the systematic proposal corrected the weights of the stocks in the portfolio, seeking to increase the return from the decrease in the participation of stocks that are being affected by the herd effect during the period of analysis. The fact that the recommended portfolio has performed better by the Mean-Variance method demonstrates a portfolio formed based on fundamentalist analysis and, therefore, is a predominantly rational portfolio, with stocks that are not being affected by the herd effect. These evidences will be proven or refuted in the statistical analysis of the results.

Further to the descriptive analysis of results, the Sharpe Index has been calculated for all portfolios in the formation period and in the analysis period, according to the optimization methods. As portfolios are formed each year from 1995 to 2010 (and analyzed over the next five years), three types of portfolios (recommended, losing and winning) optimized by five different optimization methods were $16 \times 3 \times 5 \times 2 = 480$ Sharpe Indexes calculated. As is a lot of information, the results are presented here by type of portfolio and by optimization method (Table 6).

Table 6 - Sharpe Indexes in the Formation Period and Analysis, by Portfolio Type and by Optimization Method

The "average" line yields the average result of the 16 Sharpe Indexes calculated for each cross analyzed, since the portfolios are formed from 1995 to 2010 to generate more information. The line "Amount of Best" brings the number of times in the 16 formations where one portfolio was better than the other by the Sharpe Index. The values marked in bold show the best portfolios by method and the cells marked in gray demonstrate the best method by portfolio.

OPTIMIZATION METHOD		SHARPE INDEX IN THE FORMATION PERIOD			SHARPE INDEX IN THE ANALYSIS		
		Recommended	Losing	Winning	Recommended	Losing	Winning
Equally Weighted	Average	0.1520	0.0461	**0.4540**	**0.2523**	0.1741	0.1794
	Amount of Best	0	0	16	10	3	3
Mean-Variance	Average	0.5560	0.3087	**0.7598**	**0.2281**	0.1413	0.1629
	Amount of Best	3	2	11	11	2	3
Proposed Systematic	Average	0.0755	-0.0131	**0.2970**	**0.1874**	0.1520	0.1371
	Amount of Best	4	1	11	8	4	4
Mean-Variance with Periodicity	Average	0.5560	0.3087	**0.7598**	**0.2395**	0.1586	0.1855
	Amount of Best	3	2	11	8	2	6
Proposed Systematic with Periodicity	Average	0.0755	-0.0131	**0.2970**	**0.1777**	0.1164	0.1272
	Amount of Best	4	1	11	8	3	5

Source: Own elaboration with the help of Microsoft Excel 2010

Firstly, analyzing portfolio types, if an investment portfolio was selected only considering its Sharpe Index in the formation period, the winning portfolio should be chosen, obviously because it has the highest returns in the formation, which maximizes its Index of Sharpe compared to the other two portfolios analyzed. However, when Sharpe's indexes are verified in the analysis period, the recommended portfolio is the one that demonstrates the best risk-return relationship, independent of the optimization method chosen.

Analyzing Table 6 with the perspective of optimization methods, the Mean-Variance method is obviously the best in the formation period, since its objective is precisely to maximize the Sharpe Index: maximize the return for a given level of risk, or minimize the risk for a given level of return. However, in the period of analysis, the application of the Mean-Variance method periodically proved to be more effective in terms of risk-return only for the winning portfolio. Contrary to expectations, the Recommended and Missing portfolios obtained the best Sharpe index when the stocks are equally weighted.

Regarding the quantities of best portfolios present in Table 6, these refer to how often one type of portfolio was better than the others in each of the optimization methods. While in the formation period by the equally weighted method the 16 winning portfolios proved to be the best, the other optimization methods were able to maximize the Sharpe Index of some recommended and losing portfolios in relation to the winners, but not enough. This generate the interpretation that if it is to observe only risk and return data in the formation period, the most appropriate strategy would be to invest in winning portfolios, which is contrary to Sharpe's index results. When these same portfolios with the highest Sharpe Indexes have been observed in the review period, regardless of the optimization method, the recommended portfolios showed to be the best in half or more of half the time.

The descriptive analyzes presented so far regarding Hypothesis III served as the basis for the formulation of the evidences to be confirmed or refuted by the hypothesis tests and the correlation analysis. These evidences are formulated by portfolios, since the expected results are different for each of them.

First, in relation to the recommended portfolio, since the choice of the assets was based on a fundamentalist analysis through the five-factor model of Fama and French (2015), it is interpreted that a portfolio based on an arbitrage model cannot have deviations of behavior. Therefore, the Mean-Variance method proved to be the best to optimize this kind of portfolio.

Thus in relation to the recommended portfolio, do not reject the fact that the Mean-Variance can be considered the best optimization method for this portfolio is tested by ANOVA or by the Friedman's Test (if the data are non-parametric). The fact that the recommended portfolio is a portfolio that does not have behavioral deviations can be proven through the absence of a negative and significant correlation between the average absolute deviation of the portfolio returns (CSAD) and the market return.

As for the losing portfolio, precisely because the assets present in it had the greatest losses in the year of formation, it is necessary to weight the participation of each one in the portfolio by the fundamentalist score, maximizing the participation of those stocks with low returns as a result of being undervalued, and therefore the systematic proposal without periodicity would work better for this portfolio, since the systematic proposed applied only in the formation period maximizes the fundamentalist score, but does not seek to correct deviations of behavior. Because these stocks were undervalued, they would not have deviations of behavior, so the annual application of the systematic proposal would reduce the return.

Thus, it is expected for the losing portfolio that the ANOVA or Friedman's Test results have significant difference results between the systematic proposal applied only in the formation period and the other methods, not rejecting the fact that this would be the best method to apply in the losing portfolio.

Lastly, on the winning portfolio, it is explained that because it comprises the 15 stocks with the highest gains in the formation period, these are overvalued stocks in relation to the market and that, therefore, the arbitrators will eventually realize their profits and exit these positions. Therefore, the herd effect can occur, which consists of the late performance of the noise traders, who buy late winning stocks, prolonging the return effects, characterizing the Momentum effect. However, the arbitrators, seeing that these stocks are overvalued even more, leave the positions and realize profits, reducing prices and bringing losses to the noise traders. As the systematic proposal with periodicity brings a decision factor that seeks to identify this moment of the noise traders' action as the moment of leaving the positions, the descriptive analysis evidenced that this method is more effective in the optimization of the winning portfolio.

Therefore, it is expected that the ANOVA or Friedman's Test result will bring significant and positive differences from the systematic proposal with periodicity and other optimization methods.

The results referring to the normality of data series demonstrated that the data do not have a normal distribution and therefore are considered as non-parametric, using the Friedman's Test for all portfolios. The results of Friedman's Test for the recommended portfolio are presented in Table 7.

Table 7 - Friedman's Test of Averages Comparison between Returns of the Recommended Portfolio

It should be noted that the values presented are from the row in relation to the column: the difference in the portfolio return by the method described in the table row minus the portfolio return by the method described in the column.

RECOMMENDED PORTFOLIO		Equally Weighted	Mean-Variance	Proposed Systematic	Mean-Variance with Periodicity	Proposed Systematic with Periodicity
Equally Weighted	Statistic	Mean Rank = 3.04	-0.294	0.056	0.256	0.184
	Significance		0.188	1.000	0.404	1.000
Mean-Variance	Statistic	0.294	**Mean Rank = 3.33**	*0.350*	**0.550**	**0.478**
	Significance	0.188		*0.051*	**0.000****	**0.001****
Proposed Systematic	Statistic	-0.056	*-0.350*	Mean Rank = 2.98	0.200	0.128
	Significance	1.000	*0.051*		1.000	1.000
Mean-Variance with Periodicity	Statistic	-0.256	**-0.550**	-0.200	Mean Rank = 2.78	-0.720
	Significance	0.404	**0.000****	1.000		1.000
Proposed Systematic with Periodicity	Statistic	-0.184	**-0.478**	-0.128	0.720	Mean Rank = 2.86
	Significance	1.000	**0.001****	1.000	1.000	

Source: Own elaboration with the aid of IBM SPSS Statistics 20.0

Interpreting the results of Friedman's Test for the recommended portfolio, the highest average of accumulated returns in the quarters was when using the Mean-Variance method of optimization. However, the difference between this and the systematic proposal was not significant, nor was it related to the equally weighted method.

The differences analyzed by the test were significant only of the Mean-Variance in relation to the two methods with periodicity of rebalance of weights. Therefore, it is verified that the annual rebalancing of the weights in the recommended portfolio decreased the portfolio result. Thus, about the method to be used in the recommended portfolio, it cannot be said that Mean-Variance would be the best option, since the result did not give significant results.

The results for the losing portfolio, presented in Table 8, show that the optimization by the Proposed Systematics in the formation period brought the highest cumulative return in the quarters, but the differences are not significant in relation to the application of other methods. The only significant difference was between this systematic proposed and itself applied with annual periodicity, demonstrating that the periodic application of the systematic proposal decreases the return of portfolio.

Table 8 - Friedman's Test of Averages Comparison between Returns of the Losing Portfolio

LOSING PORTFOLIO		Equally Weighted	Mean-Variance	Proposed Systematic	Mean-Variance with Periodicity	Proposed Systematic with Periodicity
Equally Weighted	Statistic	Mean Rank = 3.05	0.122	-0.191	0.125	0.194
	Significance		1.000	1.000	1.000	1.000
Mean-Variance	Statistic	-0.122	Mean Rank = 2.93	-0.312	0.003	0.072
	Significance	1.000		0.124	1.000	1.000
Proposed Systematic	Statistic	0.191	0.312	**Mean Rank = 3.24**	0.316	**0.384**
	Significance	1.000	0.124		0.116	**0.021***
Mean-Variance with Periodicity	Statistic	-0.125	-0.003	-0.316	Mean Rank = 2.92	0.069
	Significance	1.000	1.000	0.116		1.000
Proposed Systematic with Periodicity	Statistic	-0.194	-0.072	**-0.384**	-0.069	Mean Rank = 2.86
	Significance	1.000	1.000	**0.021***	1.000	

Source: Own elaboration with the aid of IBM SPSS Statistics 20.0

The results of Table 9 (Winning Portfolio) confirm the descriptive results from the moment when significant differences have been found between the Proposed Systematics (both applied only in the formation period and with periodicity) and the other models, proving that the systematic proposal is the best model for optimize winning portfolios. However, it has not been possible to conclude if the proposed systematic periodicity is better than the systematic proposal applied only in the formation period.

Finishing this subtopic of analysis, we try to answer the following question: Which optimization model is best for each portfolio? According to the results of Friedman's tests, in the case of recommended and losing portfolios, there are no significant differences between the models. Therefore, we opted for the equally weighted method, which is the one that generates the lowest transaction cost and the best risk-return relationship, according to the Sharpe Index data. Regarding the winning portfolio, it is still not possible to reach a conclusion, since the Friedman's test points to the Proposal Systematics as the best, but the Sharpe Index shows that, on average, the best relation between risk and return is found when the winning portfolios are optimized by Mean-Variance on an annual basis.

Table 9 - Friedman's Test of Averages Comparison between Returns of Winning Portfolio

WINNING PORTFOLIO		Equally Weighted	Mean-Variance	Proposed Systematic	Mean-Variance with Periodicity	Proposed Systematic with Periodicity
Equally Weighted	Statistic	Mean Rank = 2.90	**0.450**	**-0.625**	*0.322*	**-0.631**
	Significance		**0.003****	**0.000****	*0.100*	**0.000****
Mean-Variance	Statistic	**-0.450**	Mean Rank = 2.45	**-1.075**	-0.128	**-1.081**
	Significance	**0.003****		**0.000****	1.000	**0.000****
Proposed Systematic	Statistic	**0.625**	**1.075**	**Mean Rank = 3.53**	**0.947**	-0.006
	Significance	**0.000****	**0.000****		**0.000****	1.000
Mean-Variance with Periodicity	Statistic	*-0.322*	0.128	**-0.947**	Mean Rank = 2.58	**-0.953**
	Significance	*0.100*	1.000	**0.000****		**0.000****
Proposed Systematic with Periodicity	Statistic	**0.631**	**1.081**	0.006	**0.953**	**Mean Rank = 3.53**
	Significance	**0.000****	**0.000****	1.000	**0.000****	

Source: Own elaboration with the aid of IBM SPSS Statistics 20.0

To solve this doubt, the Friedman's Test was also applied comparing the Sharpe Indexes of the winning portfolios with the five optimization methods. It is expected that the differences between them will not be significant, and will then opt for the Proposed Systematics as a model to optimize the winning portfolios.

Table 10 - Friedman's Test Between the Sharpe Indexes of the Winning Portfolio

Test Statistic	3,850
Degrees of freedom	4
statistical significance	0.427

Source: Own elaboration with the aid of IBM SPSS Statistics 20.0

According to the results of Table 10, the differences between the Sharpe Indexes of the winning portfolio are not significant between them. Therefore, it is concluded that because the difference between the Sharpe indexes is not significant and the difference of the returns by the optimization methods is significant, we opted for the Proposed Systematics method to optimize winning portfolios. However, it is not possible to conclude whether it is best to apply the method only in the formation period or periodically during the analysis. As the differences between them are not significant, we choose to apply the Proposed Systematics only in training, since it generates lower transaction costs.

5. CONCLUSIONS

As described in the introduction, the research question on which this study was based was: " Does a systematic optimization of portfolios that accompany the dynamics of the herd effect and the overconfidence in the Brazilian Capital Market can increase the return on investment in relation to Markowitz's (1952) method of portfolio selection? "

The results obtained in descriptive terms showed that the recommended portfolio would have a better return if applied the optimization by Mean-Variance only in the formation period, corroborating with the fact that it is an essentially fundamentalist portfolio. However, the hypothesis tests showed that the differences between the models were not significant, making the recommended portfolio when equally weighted more efficient than the other methods.

In the case of losing portfolios, in terms of descriptive results of the return, the systematic proposal would be the best. This is logical from the moment that the systematic proposal maximizes, among the losers, the weight of those more undervalued in relation to its fair price. However, the statistical tests were not significant, and it was also concluded that the equally weighted portfolio is the best because it reduces transaction costs.

In the case of the winning portfolio, the systematic proposal both in the formation period and periodically showed a very positive return and above the other optimization methods. These results were confirmed by the Friedman’s Test, being insignificant only the relationship between the proposed systematic in the formation period and periodically. As the systematic proposal applied only in the formation period tends to have lower transaction costs, this method was initially chosen.

Sharpe's indexes of portfolios resulted in that the method of Mean-Variance on a periodic basis would be better in terms of risk and return for the winning portfolio. Statistically analyzing the difference between the Sharpe indexes calculated by the five optimization methods analyzed, this was not significant and was therefore chosen the systematic method proposed in the formation period as the most suitable method for the winning portfolio.

The results obtained corroborate with Bohm and Wenzelburger (2005), which demonstrated that rational investors' portfolios in the mean-variance plan are considered efficient ex-ante (portfolio formation) when compared to noise traders. However, the authors have shown that there is no direct relationship between their ex-ante efficiency and their ex-post empirical performance (as measured by the Sharpe Index), and that noise traders may have better ex-post performance than investors who base the choice of their portfolio in the efficient mean-variance plan. That is, it is possible to obtain larger returns with other models than Mean-Variance.

In this way, complementing the analysis made by Grinblatt, Titman and Wermers (1995) and corroborating with Cronqvist and Siegel (2014) and with Jegadeesh and Titman (2011), there is the individuals' propensity to choose titles that performed well in the recent past, causing the Momentum, being the Momentum here explained by the herd effect.

The fact that the difference between the method of optimization by the Mean-Variance and the systematic proposal was not significant in the recommended and losing portfolios corroborates with the result of Das et. al. (2010), because they have proven that efficient sub-portfolio combinations in the mean-variance plan result in an efficient aggregate portfolio and that, since the approach to mental accounting is mathematically equivalent to that of the mean-variance, mental accounting-based subportfolios are also efficient in the mean-variance plane. In other words, the method proposed here is as efficient as Mean-Variance, and it is not possible to say which method is the best.

Relating the results to the level of the Brazilian market efficiency, Nakamura (2000) and more recently Noda, Martelanc and Securato (2014) did not reject the Brazilian capital market efficiency hypothesis (measured by the Ibovespa and other indexes and their relationship to a reference portfolio). In other words, since the market is relatively efficient, in most of the analyzed portfolios (recommended and losing) returns for the systematic proposal were not the best because they are essentially fundamentalist portfolios and are the ones that already provide the highest returns. The systematic proposal also considers the maximization of the fundamentalist score of the winning portfolio in its formation and, therefore, it is concluded that, in general, the Brazilian capital market is rational, with the presence of deviations of behavior in specific stocks and probably in less periodicities.

Therefore, it is possible to affirm that the general objective of developing a systematic optimization of portfolios that accompanies the dynamism of the market anomalies present in the Brazilian capital market was met.

This paper concludes by presenting its limitations and perspectives for future studies. After the development of the systematic proposal, it is possible to suggest an improvement in this systematics as part of future research: periodic optimization calculates the weights in order to maximize a score that is composed by the decision factor of maintaining, decreasing or increasing the participation of a stock and fundamentalist points of them. But when does that stock still remain undervalued and the fundamentalist score continues to decline as returns increase? It has not yet been possible to diagnose this mid-term decision point between whether or not the score will continue to decline from one period to the next.

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